Decision Making of Networked Multiagent Systems for Interaction Structures

Yichuan Jiang, Member, IEEE, Jing Hu, and Donghui Lin

Abstract—Networked multiagent systems are very popular in large-scale application environments. In networked multiagent systems, the interaction structures can be shaped into the form of networks where each agent occupies a position that is determined by such agent's relations with others. To avoid collisions between agents, the decision of each agent's strategies should match its own interaction position, so that the strategies available to all agents are in line with their interaction structures. Therefore, this paper presents a novel decision-making model for networked multiagent strategies based on their interaction structures, where the set of strategies for an agent is conditionally decided by other agents within its dependence interaction substructure. With the presented model, the resulting strategies available to all agents can minimize the collisions of multiagents regarding their interaction structures, and the model can produce the same resulting strategies for the isomorphic interaction structures. Furthermore, this paper uses a multiagent citation network as a case study to demonstrate the effectiveness of the presented decision-making model.

Index Terms—Citation networks, decision making, multiagents, networked interaction structures, social network analyses.

I. INTRODUCTION

S TRATEGY is the action that an agent adopts to behave in multiagent systems [1]; for example, an agent can select the strategy of cooperation or defect in the Prisoner's

Manuscript received January 25, 2010; revised May 16, 2010; accepted September 9, 2010. Date of publication March 10, 2011; date of current version October 19, 2011. This work was supported in part by the National Natural Science Foundation of China under Grant 60803060, by the National High Technology Research and Development Program of China (863 Program) under Grant 2009AA01Z118, by the Specialized Research Fund for the Doctoral Program of Higher Education of the State Education Ministry of China under Grants 200802861077 and 20090092110048, by the General Program of Humanities and Social Sciences in University of the State Education Ministry of China under Grant 10YJCZH044, by the Program for New Century Excellent Talents in University of the State Education Ministry of China under NCET-09-0289, by the Open Research Fund from the Key Laboratory of Computer Network and Information Integration in Southeast University, State Education Ministry of China, under Grant K93-9-2010-16, by the State Key Laboratory for Manufacturing Systems Engineering (Xi'an Jiaotong University) under Grant 2010001, and by the NUAA Research Funding under Grant NR2010001. This paper was recommended by Associate Editor F. Gomide.

Y. Jiang is with the Key Laboratory of Computer Network and Information Integration of State Education Ministry, School of Computer Science and Engineering, Southeast University, Nanjing 211189, China, and also with the State Key Laboratory for Manufacturing Systems Engineering, Xi'an Jiaotong University, Xi'an 710054, China (e-mail: jiangyichuan@yahoo.com.cn; yjiang@seu.edu.cn).

J. Hu is with the College of Arts, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China (e-mail: njuhj@nuaa.edu.cn).

D. Lin is with the National Institute of Information and Communications Technology, Tokyo 184-8795, Japan (e-mail: lindh@nict.go.jp).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSMCA.2011.2114343

Dilemma game. When agents are uncertain about the precise results of adopting such strategies, they will take decision making as a means to analyze which of a series of strategies should be taken [2], [34]. Previous works on decision making always use game theory or other economics techniques, such as negotiation, bargaining, auction, and contracting, and mainly concern negotiation protocols and decision-making procedures [3], [4], [34].

Nowadays, with the development of large-scale multiagent systems, agents are always organized in networked structures where an agent interacts only with its immediate neighbors [5]-[7]. The interactions among agents can be shaped in the form of networks in which the vertices denote the agents and the edges indicate their interaction relations. According to the social-network-analysis method, each agent occupies a position against other agents in the interaction structure [8]; such position can only be shaped and described by the agent's relations with other agents, but not by its own inherent attributes. The interaction position of an agent is very important to determining its role in the system [9]; for example, in a multiagent system simulating corporation organization, the action strategies of a manager agent should be different from those of a staffer agent. If the staffer agent adopts the manager agent's strategies, some disorders may result. Therefore, agents should adopt the strategies that match their positions in the interaction structures to avoid colliding with other agents. Making decisions according to the interaction structure is a crucial problem in solving agent coordination within networked multiagent systems.

Distributed decision making for the coordination of networked agents has received much attention in recent years. Saber and Murray [10] provide convergence, performance, and robustness analyses of an agreement protocol for a network of integrator agents with directed information flow and switching topology, which mainly rely on the tools of algebraic graph theory and matrix theory. Roy et al. [11] introduce a quasilinear stochastic distributed protocol that can be used by a network of sensing agents to reach a collective action agreement. Generally, previous works on the decision making of networked agents mainly concerned the agreement problems in which all agents within the network must achieve the same opinion, and the connection between the networked topology and the performance of the agreement protocol. Therefore, previous works seldom took into account the interaction structures of agents while they decide the agents' action strategies.

In addressing the aforementioned issues, this paper takes both the agent's interaction position and the strategies into account and presents a model for deciding the strategies available to agents that can satisfy the constraints created by interaction *structures*. In our decision-making model, we focus upon the multiagent dependence relations that are always seen where the strategies of some agents are dependent on others [12]. Dependence relations occur in multiagent systems due to many reasons, such as resource constraints, environmental constraints, task allocations, etc. Clearly, the strategies available to an agent should be conditionally determined by its dependent agents within the interaction structure.

In this paper, the main purpose of decision making is to put restrictions on agent strategies that can match the interaction positions of all agents within the interaction structure. With our approach, the decision-making result can minimize the collisions between agents regarding their interaction positions.

The rest of this paper is organized as follows. In Section II, we model the interaction structures of networked multiagents; in Section III, we present the decision-making model for interaction structures; in Section IV, we highlight a case study of decision-making in multiagent citation networks; in Section V, we introduce related work; and finally, we conclude this paper in Section VI.

II. MODELING THE INTERACTION STRUCTURES IN NETWORKED MULTIAGENT SYSTEMS

A. Interaction Structures

An interaction structure consists of the interaction relations between agents. In this paper, we summarize the interaction relations to be in two forms: 1) the dependence relations, which denote that some agents are reliant on other ones; and 2) the domination relations, which denote that some agents can overpower others. For an agent, the "*in*" interaction relations denote its dependence on other agents and the "*out*" interaction relations refer to its domination over other agents.

Generally, the interaction structure among agents can be understood in a networked form in which the vertices denote the agents and the edges symbolize their interaction relations. According to the network analysis method [8], each agent has its position against other agents in the interaction network. Clearly, such a position can only be shaped and described by the agent's relations with other agents, but not by the inherent attributes of an individual agent.

Definition 1: The interaction position of an agent is defined as the set of interaction relations of various types linking this agent with other agents. Let the agent interaction structure be $N = \langle A, R \rangle$, where A denotes the set of agents and R represents the set of agent interaction relations. The position of an agent a_i is the union of its immediate *in* interaction relations and immediate *out* interaction relations

$$P_{a_i} = \{ \langle a_i, a_j \rangle | a_j \in A \land \langle a_i, a_j \rangle \in R \} \\ \cup \{ \langle a_j, a_i \rangle | a_j \in A \land \langle a_j, a_i \rangle \in R \}.$$
(1)

Example 1: Fig. 1 shows an agent interaction structure and the positions of agents a, c, e, f, h, and k.

If agent a is the source of an interaction relation $r \in R$, then we can refer to it as $a \odot r$. If agent a is the destination of an interaction relation $r \in R$, then we can denote it as $a \otimes r$.



Fig. 1. Example of multiagent interaction structure and agent interaction positions.

Definition 2: The dependence substructure of an agent in the interaction structure is defined as the set of in interaction relations of various types linking this agent with other agents. Let the agent interaction structure be $N = \langle A, R \rangle$, where A denotes the set of agents and R denotes the set of agent interaction relations. Then, the *first-order dependence substructure of an agent*, namely, $a_i \in A$, is the union of its immediate *in* links

$$Dep_{a_i} = \{ \langle a_j, a_i \rangle | a_j \in A \land \langle a_j, a_i \rangle \in R \}.$$
⁽²⁾

Obviously, the *second-order dependence substructure* of a_i can be defined as

$$Dep(Dep_{a_i}) = \{ \langle a_k, a_j \rangle | a_j \in A \land a_k \in A \\ \land \langle a_j, a_i \rangle \in R \land \langle a_k, a_j \rangle \in R \}.$$
(3)

Therefore, the *n*th-order dependence substructure of a_i can be defined as

$$\prod_{n} Dep_{a_{i}} = \underbrace{Dep\left(Dep\left(\cdots\left(Dep_{a_{i}}\right)\cdots\right)\right)}_{n} = \left\{\langle a_{n}, a_{n-1}\rangle | a_{1} \in A \land a_{2} \in A \land \cdots \right. \\ \wedge a_{n} \in A \land \langle a_{n}, a_{n-1}\rangle \in R \land \cdots \\ \wedge \langle a_{2}, a_{1}\rangle \in R \land \langle a_{1}, a_{i}\rangle \in R \right\}.$$
(4)

The set of agents within the first-order dependence substructure of a_i (called *its first-order dependence agents*) is

$$\begin{aligned}
\mathfrak{V}_{a_i} &= \{a_j | a_j \in A \land \langle a_j, a_i \rangle \in Dep_{a_i} \} \\
&= \{a_j | a_j \odot r \land r \in Dep_{a_i} \}.
\end{aligned}$$
(5)

Therefore, the set of all agents within the all-orders dependence substructures of agent a_i is

$$\sum \mathfrak{V}_{a_i} = \bigcup_k \left\{ a_j | a_j \odot r \land r \in \prod_k Dep_{a_i} \right\}.$$
(6)

On the other hand, an agent may also influence other agents' strategies for actions within the interaction structure, so we have the following definition.

Definition 3: The domination substructure of an agent in the interaction structure is defined as the set of out interaction relations of various types linking this agent with other agents. Let the agent interaction structure be $N = \langle A, R \rangle$, where A denotes the set of agents and R denotes the set of agent interaction relations. Then, the first-order domination substructure of an agent, namely, $a_i \in A$, is the union of its immediate out links

$$Dom_{a_i} = \{ \langle a_i, a_j \rangle | a_j \in A \land \langle a_i, a_j \rangle \in R \}.$$
(7)



Fig. 2. Dependence and domination substructures of agent d.

Obviously, the second-order domination substructure of a_i can be defined as

$$Dom (Dom_{a_i}) = \{ \langle a_j, a_k \rangle | a_j \in A \land a_k \in A \\ \land \langle a_i, a_j \rangle \in R \land \langle a_j, a_k \rangle \in R \}.$$
(8)

Therefore, the *n*th-order domination substructure of a_i can be defined as

$$\prod_{n} Dom_{a_{i}} = \overbrace{Dom \left(Dom \left(\cdots \left(Dom_{a_{i}} \right) \cdots \right) \right)}^{n}$$
$$= \{ \langle a_{n-1}, a_{n} \rangle | a_{1} \in A \land a_{2} \in A \land \cdots$$
$$\land a_{n} \in A \land \langle a_{i}, a_{1} \rangle \in R \land \langle a_{1}, a_{2} \rangle \in R \land \cdots$$
$$\land \langle a_{n-1}, a_{n} \rangle \in R \}.$$
(9)

The set of agents within the first-order domination substructure of a_i (called *its first-order domination agents*) is

$$\Omega_{a_i} = \{a_j | a_j \in A \land \langle a_i, a_j \rangle \in Dom_{a_i} \}$$
$$= \{a_j | a_j \otimes r \land r \in Dom_{a_i} \}.$$
(10)

Therefore, the set of all agents within the all-orders domination substructures of agent a_i is

$$\sum \Omega_{a_i} = \bigcup_k \left\{ a_j | a_j \otimes r \wedge r \in \prod_k Dom_{a_i} \right\}.$$
(11)

Example 2: Now, we consider the dependence and domination substructures of agent *d* in Fig. 1, as shown in Fig. 2

$$Dep_{d} = \{\langle a, d \rangle, \langle c, d \rangle\} \qquad \prod_{2} Dep_{d} = \{\langle b, a \rangle, \langle b, c \rangle\}$$
$$\mathcal{O}_{d} = \{a, c\} \qquad \sum \mathcal{O}_{d} = \{a, c, b\} \qquad Dom_{d} = \{\langle d, e \rangle\}$$
$$\prod_{2} Dom_{d} = \{\langle e, j \rangle\} \qquad \prod_{3} Dom_{d} = \{\langle j, h \rangle, \langle j, i \rangle, \langle j, k \rangle\}$$
$$\prod_{4} Dom_{d} = \{\langle i, k \rangle\} \qquad \Omega_{d} = \{e\}, \sum \Omega_{a_{i}} = \{e, j, h, i, k\}.$$

Lemma 1: Let an agent interaction structure be $N = \langle A, R \rangle$. If N is a directed acyclic graph (DAG), we have $\forall a, b \in A$, $a \in \sum \Omega_b \Rightarrow a \notin \sum \mho_b$ and $a \in \sum \mho_b \Rightarrow a \notin \sum \Omega_b$. *Proof:*

1) If $\exists a, b \in A \Rightarrow a \in \sum \Omega_b \land a \in \sum \mho_b$, $a \in \sum \Omega_b$ denotes that there is a path from b to a and $a \in \sum \mho_b$

denotes that there is a path from *a* to *b*; therefore, there is a cycle which contains *a* and *b*.

If ∃a, b ∈ A ⇒ a ∈ ∑ 𝔅_b ∧ a ∈ ∑ Ω_b, a ∈ ∑ 𝔅_b denotes that there is a path from a to b and a ∈ ∑ Ω_b denotes that there is a path from b to a; hence, there is a cycle that contains a and b.

Obviously, those situations are impossible in a DAG; therefore, we have Lemma 1. \Box

If two agents have identical ties to and from all other agents in the interaction structure, we can say that they are structurally equivalent.

Definition 4: Interaction structural equivalence[8]. Let the agent interaction structure be $N = \langle A, R \rangle$, where A denotes the set of agents and R denotes the set of agent interaction relations; |A| = m, |R| = n, $a_i, a_j \in A$, and $1 \le i, j \le m$. Then, a_i and a_j are structurally equivalent if for all agents $a_k \in A$, $k = 1, \ldots, m$ and $k \ne i, j$, and all interaction relations $r_x, x = 1, \ldots, n, a_i$ has an interaction relation to a_k , if and only if a_j also has an interaction relation to a_k , and a_i has an interaction relation from a_k . If a_i and a_j are structurally equivalent, we can denote them as $a_i \equiv a_j$.

Lemma 2: If two agents are structurally equivalent, then they have the same first-order dependence and domination agents, i.e., $(a_i \equiv a_j) \Rightarrow (\mho_{a_i} = \mho_{a_j}) \land (\Omega_{a_i} = \Omega_{a_j})$. Moreover, they also have the same *n*th-order (n > 1) dependence and domination agents.

Proof: From Definition 4, if two agents are structurally equivalent, they have the same first-order dependence agents and first-order domination agents. According to (4) and (9), the *n*th-order (n > 1) dependence substructure is fully dependent on the (n - 1)th-order dependence agents and the *n*th-order (n > 1) domination substructure is fully controlled by the (n - 1)th-order dependence agents. Therefore, the two agents have the same *n*th-order (n > 1) dependence and domination agents.

Example 3: From Fig. 1, the agent sets that have immediate *in* interaction relations to *a* and *c* are the same, namely, $\{b\}$, and the agent sets that have immediate *out* interaction relations from *a* and *c* are also the same: $\{d, e\}$. Therefore, agents *a* and *c* are structurally equivalent. Moreover, *h* and *k* are also structurally equivalent. Obviously, $\mho_a = \mho_c = \{b\}$, $\Omega_a = \Omega_c = \{d, e\}$, $\mho_h = \mho_k = \{i, j\}$, $\Omega_h = \Omega_k = \{\}$, $\sum \mho_a = \sum \mho_c = \{d, e\}$, $\sum \Omega_a = \sum \Omega_c = \{d, e, j, h, i, k\}$, and $\sum \mho_h = \sum \mho_k = \{i, j, g, e, a, c, d, b, f\}$; therefore, Lemma 2 is validated.

Definition 5: If agent a is not in the all-orders dependence and domination structures of agent b, i.e., $(a \notin \sum \mho_b) \land (a \notin \sum \Omega_b)$ is true, then we can think that agent a is independent from agent b, which can be denoted as $a \not\models b$. Obviously, we have $\forall a, b \in A, a \not\models b \Rightarrow b \not\models a$.

Lemma 3: Let an agent interaction structure be $N = \langle A, R \rangle$; if N is a DAG, we have $\forall a, b \in A, a \equiv b \Rightarrow a \not\models b$.

Proof: Let $\exists a, b \in A, a \equiv b$. If $\neg(a \not\geq b)$ is true, then $(a \in \sum \Omega_b) \lor (b \in \sum \Omega_a)$ is true. Now, $a \equiv b$; hence, $(\sum \mho_a = \sum \mho_b) \land (\sum \Omega_a = \sum \Omega_b)$ is true, which denotes that $(a \in \sum \Omega_a) \lor (b \in \sum \Omega_b)$ is true. Such situation is impossible in a DAG. Therefore, we have Lemma 3.

B. Constraints Among Agents' Strategies in the Interaction Structures

When agents interact with and depend on each other, there may be some constraints which limit their available strategies for avoiding collisions. For example, if there are two paths between places x and y and each path can only be passed by one agent at the same time, agent a_1 will go from x to y and agent a_2 will go from y to x. Moreover, a_1 has the priority to select the path first (i.e., the decision of strategy of a_2 depends on the strategy of a_1). Thus, we can set the constraint to a_2 as " a_2 cannot select the same path as a_1 ".

Definition 6: Social constraint. Let there be, first, a finite set of agents, $A = \{a_1, a_2, \ldots, a_n\}$, and second, an initial set of strategies for each agent, containing a finite and discrete strategies' domain for each agent, $S = \{S_1, S_2, \ldots, S_n\} \forall i \in$ [1, n] and $s_{ij} \in S_i$, where s_{ij} is the *j*th strategy that agent a_i adopts in the operation. Then, a social constraint set is $C = \{C(A_1), C(A_2), \ldots, C(A_m)\}$, where each A_i is a subset of the agents and each social constraint $C(A_i)$ is a set of tuples indicating the mutually consistent strategy values of the agents in A_i .

In reality, the social constraint with the arity of 2 is always seen and is the basic form of most constraints. Thus, we mainly consider such constraint form in this paper.

Definition 7: A binary social constraint is the one that only affects two agents. If there is a binary social constraint c_{ij} occurring from agent a_i to a_j , i.e., the constraint is only endowed on a_j by a_i , we can say that such a constraint is unilateral. Now, agent a_i is referred to as the subject one of c_{ij} and a_j is called the object one of c_{ij} .

Therefore, if a social constraint is unilateral, we think that the strategies of the object agent will be influenced by the subject agent, but the strategies of the subject agent will not be influenced by the object agent. Obviously, the dependence relation between two agents can be considered as a unilateral binary social constraint between them; thus, we can use the unilateral binary social constraint between two agents to denote the interaction relation between them. Let there be two agents, a_i and a_j ; if a_j depends on a_i , then the available strategies of a_j are constrained by a_i .

In real systems, some social constraints can be satisfied, but others can never be satisfied; accordingly, some dependence relations can be fulfilled, but some other ones cannot be satisfied in the interaction structure. Therefore, we have the following definition.

Definition 8: Lawful and unlawful dependence relations. Let there be a dependence constraint c_{ij} occurring from agents a_i to a_j . The sets of strategies for a_i and a_j are S_i and S_j , respectively. If $\forall s_j \in S_j$, there exists a strategy in S_i , i.e., $\exists s_i \in S_i$, that satisfies c_{ij} , we can say that c_{ij} is lawful. Otherwise, we say that c_{ij} is unlawful. If S_j is empty, we can also say that this dependence relation is unlawful.

If an agent is restricted by more constraints, then its behavior freedom will be more limited.

Definition 9: Freedom degree of agent. For an agent a_j , let the initial set of strategies for a_j while it is not constrained by any dependence relations be S; now, if a_j is constrained by the set of dependence relations $\cup_i c_{ij}$, then S' is the set of strategies of a_j that take into account the dependence relations $\cup_i c_{ij}$. Therefore, the freedom degree of agent a_j under the dependence relations $\cup_i c_{ij}$ is

$$f_j = |S'| / |S|.$$
(12)

If S' is empty, we say that the freedom degree of a_j is 0.

III. DECISION-MAKING MODEL FOR MULTIAGENT INTERACTION STRUCTURES

On the basis of our previous work on the social law extracting model in networked multiagent systems [31], here, we present a decision-making model for multiagent interaction structures.

A. Some Concepts

The conditional dependence assumption provides a practical way to construct the joint distribution among agents [13]. As stated in Section II, the strategies of some agents are conditionally determined by their dependence agents.

Definition 10: Conditional strategy. An interaction structure $\langle A, R \rangle$ is given, where the first-order dependence structure of agent $a \in A$ is Dep_a and the set of strategies available to the agents of Dep_a is $\bigcup_{i \in \mathcal{O}_a} S_i$. Therefore, the set of strategies available to a is the one given that the agents of its dependence substructure adopt the strategies $\bigcup_{i \in \mathcal{O}_a} S_i$; this can be referred to as the conditional strategy $S_{a|\mathcal{O}_a}$.

The different constraints of dependence relations may produce collisions among themselves. To achieve the global harmony of the system, we should implement decision making to restrict the agents' strategies so as to minimize the conflicts among agents (the definition of decision making in this paper is a little similar to the concept of social law in [14] and [15]).

Definition 11: Decision of multiagent strategies. An environment $\langle A, S_1, S_2, \ldots, S_n \rangle$ is given, where A is the set of agents, $A = \{a_1, a_2, \ldots, a_n\}$, and S_i is the initial set of strategies for a_i . We define a decision of multiagent strategies to be a restriction of S_1 to $S_1^* \subseteq S_1, S_2$ to $S_2^* \subseteq S_2, \ldots, S_n$ to $S_n^* \subseteq S_n$, which can minimize conflicts among agents.

Definition 12: Useful decision of multiagent strategies. Let the total constraints of dependence relations in the system be C; a decision $SL = \langle S_1^*, S_2^*, \ldots, S_n^* \rangle$ is completely useful if for every constraint of dependence relation, $\forall c_{ij} \in C$, there exists $s_i \in S_i^*$ such that $\forall s_j \in S_j^*$, we have $\langle s_i, s_j \rangle$ satisfying c_{ij} . Therefore, in a completely useful decision, all dependence relations are lawful.

However, in real multiagent systems, sometimes, a decision may only satisfy the requirements of some but not all dependence relations, i.e., a decision may be only *approximately* useful. Then, how could one evaluate the usefulness of a decision for the interaction structure? We can use the following definition.

Definition 13: Usefulness degree of a decision. Let the set of constraints of dependence relations that can be satisfied by the decision SL be C_{sat} , and if the total constraints of dependence relations in the system is C, then at first, we can simply define



| Strategy profiles of agents | | | | Social constraints | | | | п |
|-----------------------------|-------------------------|-------------------------------------|--------------------------------|--------------------|-----------------|--------------|--------------|-----------------|
| a_1 | <i>a</i> ₂ | a3 | a_4 | C ₁₂ | C ₂₄ | C13 | C43 | u |
| $\{1,,10\}$ $f_1=1$ | $\{1,,10\}$ $f_2=1$ | {1,,10} <i>f</i> ₃ =1 | $\{1,10\}$ $f_4=1$ | \checkmark | × | × | × | $\frac{5}{8}$ |
| $\{1,3,5,7\}$ $f_1=0.4$ | $\{1,3,5,7\}\ f_2=0.4$ | $\{1,3,5,6\}$ $f_2=0.4$ | $\{1,3,4\}$ $f_2=0.3$ | \checkmark | \checkmark | × | × | $\frac{7}{16}$ |
| $\{5,6,8,9\}$ $f_1=0.4$ | $\{3,5,10\}\ f_1=0.3$ | $\{2,3,5,7\}$ $f_1=0.4$ | $\{1,3\}$ $f_1=0.2$ | × | \checkmark | × | × | $\frac{23}{80}$ |
| $\{1,,10\}$ $f_1=1$ | $\{1,,9\}$ $f_1=0.9$ | $\{1,, 4\}$ $f_1=0.4$ | $\{1, \dots, 6\}$ $f_1=0.4$ | \checkmark | \checkmark | \checkmark | \checkmark | $\frac{67}{80}$ |

Remarks: α =0.5, β =0.5. $\sqrt{}$: lawful, ×: unlawful. *U*: usefulness degree. (b)

Fig. 3. Example for strategy constraints and decision making.

the usefulness degree of a decision SL as $|C_{\text{sat}}|/|C|$. Moreover, a decision should enable the agents to have more total freedom under the condition that social constraints can be satisfied. Thus, we can extend the definition as

$$U_{\rm SL} = \alpha \left(\left(\sum_{i=1}^{n} f_i \right) / n \right) + \beta \left(|C_{\rm sat}| / |C| \right)$$
(13)

where α and β are parameters to define the relative importance of agent freedom degree and dependence constraint satisfaction degree, respectively. We can set the values of the parameters according to real situations. Therefore, our aim of decision making is to explore the decision with the maximum degree of usefulness.

Example 4: Fig. 3 is an example of the decided multiagent strategies with different usefulness degrees. Fig. 3(a) is an interaction structure where there are four agents $\{a_1, a_2, a_3, a_4\}$ and four social constraints taken by the dependence relations $\{c_{12} : s_1 > s_2, c_{24} : (s_2 - 2) > s_4, c_{13} : (s_1 - 5) > s_3, c_{43} : s_4 > s_3\}$. In such a system, it is assumed that the agents can take actions of adopting values in the set $S = \{1, 2, \ldots, 10\}$; hence, the strategies are the values in S. The initial strategy profiles of four agents are all set to $\{1, 2, \ldots, 10\}$. Now, we randomly make four decisions and compute their usefulness degrees, as shown in Fig. 3(b).

B. Basic Decision-Making Model for DAG Structures

If an interaction structure $N = \langle A, R \rangle$ is a DAG, then we have the following: $\forall a_i \in A$, \mho_{a_i} is fixed [16]. The basic idea in our model is as follows: $\forall a_i \in A$, if we want to decide the set of available strategies for agent a_i , we should decide the sets of available strategies for $\forall a_i \in \mho_{a_i}$ in advance. Thus, we can obtain the joint distribution of strategies for all agents step by step.

Therefore, our algorithm can be designed as follows: At first, we restrict the available strategies of the agents whose dependence agents are all decided or empty; such iteration will be repeated until it cannot find any undecided agents whose dependence ones are all decided or empty. Now, if all agents in the system can be decided with definite strategies, then the decision making is successful; otherwise, it can be noted that there are cycles in the interaction structure.

Algorithm 1. Decision making of multiagent strategies for directed acyclic interaction structure

- Input $A = \{a_1, a_2, ..., a_n\}$ and R;
- Input $S = \{S_1, S_2, \dots, S_n\}$; /* the initial strategies */
- Creatstack (stack);
- For $\forall a_i \in A$:

if
$$\mathcal{O}_{a_i} = \{\}$$
, push $(a_i, \text{ stack})$

• $A' = \{ \};$

- *While* (!empty(stack)) *do*:
 - 1) $a_u = pop(\text{stack});$
 - 2) $A' = A' \cup \{a_u\};$
 - 3) for agent ∀a_j ∈ Ω_{a_u}do:
 i) Restrict S_j according to c_{uj};
 ii) U_{a_i} = U_{a_i} {a_u};

iii) if
$$\tilde{U}_{a_i} = \{\}$$
, push $(a_i, \text{ stack});$

- If A == A', return ("There are no cycles"); else return ("There are cycles");
- Output $S_{a_i} \forall a_i \in A'$.

Algorithm 1 is $O(n^*e)$, where *n* denotes the number of agents and *e* denotes the number of dependence relations.

Theorem 1: Let the interaction structure be $N = \langle A, R \rangle$. If N is a DAG, then Algorithm 1 can make a unique decision, i.e., the decided strategies for all agents are definite.

Proof: $\forall a_i \in A$, the set of available strategies for a_i in the decision making is fully determined by the following three factors:

- 1) the initial set of strategies for a_i ;
- 2) the set of social constraints using a_i as object agent, namely, Dep_{ai} ;
- 3) the set of available strategies for \mho_{a_i} in the decision making.

Obviously, the uniqueness of 1) can be satisfied. Now, the core procedure of Algorithm 1 is the same as the one of the Topology Sorting Algorithm [17]; hence, \mathcal{O}_{a_i} is unique and the uniqueness of 2) and 3) can also be satisfied. Therefore, Algorithm 1 can make a unique decision for a directed acyclic structure.

Lemma 4: In the multiagent strategies decided by Algorithm 1, if the set of strategies for one agent is empty, then the dependence relations using such agent as object are unlawful.

Proof: From Definition 8, we have Lemma 4. \Box

Two graphs containing the same number of graph vertices connected in the same way are considered isomorphic [18], [19]; now, we present the definition of isomorphic multiagent interaction structures, shown as follows.

Definition 14: Let there be two interaction structures; one is G with the agent set $A_g = \{a_{g1}, \ldots, a_{gn}\}$, and the other is H with the agent set $A_h = \{a_{h1}, \ldots, a_{hn}\}$. G and H are said to be *isomorphic* if, first, there is a bijection f such that interaction



Fig. 4. Directed cyclic dependence structure.

relation $\langle a_i, a_j \rangle$ is in G iff $\langle f(a_i), f(a_j) \rangle$ is in H, and second, the constraints taken by the interaction relations $\langle a_i, a_j \rangle$ and $\langle f(a_i), f(a_j) \rangle$ are the same.

Theorem 2: Assume a scenario in which two interaction structures G and H satisfy the following: 1) G and H are both DAGs; 2) G and H are isomorphic; and 3) $\forall a_{gi} \in A_g$ and its peer in H, namely, $f(a_{gi})$, the initial strategies of a_{gi} and $f(a_{gi})$ are the same. Then, we can deduce that the decided strategies of a_{gi} and $f(a_{gi})$ by using Algorithm 1 are the same.

Proof: From the definitions of conditional strategy and decision making in the interaction structure, for agent a, it is determined by its first-order dependence structure Dep_a . Therefore, a's final strategies in the restriction of decision making is determined by the following: 1) the agents in Dep_a (i.e., \mathcal{V}_a); 2) the strategies of \mathcal{V}_a ; and 3) the constraints taken by the interaction relations from $\forall a_j \in \mathcal{V}_a$ to a. Now, while Algorithm 1 is used, a_{gi} and $f(a_{gi})$ have the same three factors if G and H are isomorphic; thus, the decided strategies of a_{gi} and $f(a_{gi})$ are the same.

C. Extended Decision-Making Model for Interaction Structures With Cycles

If there are any cycles in the interaction structures, then there exist some agents whose \mathcal{O}_{a_i} cannot be decided. For example, Fig. 4(a) is a directed cyclic structure, where $a_1 \in \mathcal{O}_{a_3}$, $a_3 \in \mathcal{O}_{a_2}$, and $a_2 \in \mathcal{O}_{a_1}$. Therefore, we cannot make a definite decision according to Algorithm 1.

There are many forms of cycles; among them, the simple cycle is popular. A simple cycle is one with no repeated vertices in the cycle. If there is more than one cycle, and none of them contains a vertex of another, then the cycles are independent [20].

For the reason of decision-making certainty, we have the following assumption, which assures that the decision with maximum usefulness degree can be obtained.

Assumption 1: In this paper, if there are dependence cycles in the interaction structures, the cycles are all simple and independent from each other.

In a simple dependence cycle, if we want to *compulsively* decide the strategies of an agent in the cycle a_i , then we can ignore the dependence relations of a_i on other agents in the cycle (now, a_i is a "cycle-breaking point"). After an agent is decided compulsively in a simple dependence cycle, other agents within the cycle can be decided step by step according to the basic model in Section III-B. Such a method can be seen



Fig. 5. Interaction structure including three groups.

in Algorithm 2. For example, in Fig. 4, if we first decide agent a_1 , then we can ignore the constraint c_{21} , as can be seen in Fig. 4(b); if we first decide agent a_2 , then we can ignore the constraint c_{32} , as can be seen in Fig. 4(c); if we first decide agent a_3 , then we can ignore the constraint c_{13} , as can be seen in Fig. 4(d).

Algorithm 2. Cycle breaking and decision making (A, a). /* A is the set of agents within the cycle and a is the cycle-breaking point. */

In the interaction structures with cycles that satisfy Assumption 1, the agents can be divided into three groups: 1) the agents whose strategies can be decided by Algorithm 1 (*Group 1*); 2) the agents in any simple cycle (*Group 2*); and 3) the agents that are not in any cycle but depend on some agents in cycles (*Group 3*). For example, in Fig. 5, a_1 is the agent of *Group 1*; a_2 , a_3 , and a_4 are the agents of *Group 2*; and a_5 is the agent of *Group 3*.

In our extended decision-making model for interaction structures with cycles, we use different methods to decide strategies for the three groups of agents: 1) For the first agent group, we can simply use Algorithm 1 to decide their available strategies; 2) for the second agent group, we can utilize the cycle-breaking method of Algorithm 2 for each agent in the cycle and then select the decision with the maximum usefulness degree; and 3) after the first and second agent groups are decided, then the third agent group can be decided by using Algorithm 1. The whole extended decision-making model can be shown as Algorithm 3.

Algorithm 3. Decision making of multiagent strategies for interaction structures with dependence cycles/* A denotes the whole set of agents, A' denotes the agents in Group 1, A''_i denotes the agents in cycle i, A^* denotes the agents in Group 3. */

• Calling Algorithm 1; /* After the execution of Algorithm 1, now, the set of remaining agents is $(\bigcup_i A''_i) \cup A^* */$

- For each A''_i :
 - $\{\forall a \in A''_i: Cycle breaking and decision making <math>(A''_i, a);$ /* Calling Algorithm 2 */ Output the decided strategies with the maximum usefulness degree;}
- Calling Algorithm 1 for A^{*}.

In Algorithm 3, the first and third parts are both $O(n \times e)$, where *n* denotes the number of agents in the group and *e* denotes the number of dependence relations in the group; the second part is $O(l \times m^2)$, where *l* denotes the number of cycles and *m* denotes the number of agents in a cycle. Therefore, Algorithm 3 is $O(l \times m^2)$.

As said in the proof of Theorem 1, the set of available strategies of an agent is determined by three factors. In Algorithm 3, the agents in A_i'' and A^* cannot satisfy the three factors. Therefore, the decision-making result of Algorithm 3 is not unique, i.e., different executions of Algorithm 3 may get different decisions. However, we can have the following theorem.

Theorem 3: Let an interaction structure be $N = \langle A, R \rangle$. If N has cycles and satisfies Assumption 1, then Algorithm 3 can make the decision with the maximum usefulness degree.

Proof: In Algorithm 3, the agents in N can be divided into three groups: 1) A', the agents whose strategies can be fully decided; 2) $\cup_i A''_i$, the agents that are attributed to any simple cycle; and 3) A^* , the set of agents that do not attribute to any cycle but depend on some agents in the cycles. Obviously, the available strategies of the agents in A' are decided by Algorithm 1; hence, the strategy restrictions endowed on A'are unique. The available strategies of the agents in $\cup_i A''_i$ are decided by the procedures in Algorithm 2, which can obtain the strategy restriction with the maximum usefulness degree. After the strategies of the agents in A^* are determined by Algorithm 1; therefore, the strategy restrictions endowed on A^* are unique. Therefore, the strategy restrictions endowed on A^* are unique. Therefore, the decision-making result for the whole system has the maximum usefulness degree.

Theorem 4: Assume that two interaction structures G and H satisfy the following conditions: 1) G and H are both directed graphs with cycles; 2) G and H are isomorphic; 3) the initial strategy spaces for any two peer agents in G and H are the same; and 4) for any cycle-breaking point in G, i.e., a, its peer agent f(a) is also the cycle-breaking point in H while we use the extended model and vice versa. Then, the following is true: For each agent in G, namely, a_i , and its peer agent in H, i.e., $f(a_i)$, the decided strategies of a_i and $f(a_i)$ by using the extended model are the same.

Proof: From the definitions of conditional strategy and decision making in the interaction structure, agent a is decided by its first-order dependence structure of agent Dep_a . Therefore, a's final strategies is decided by the following: 1) the agents in Dep_a (i.e., \mathcal{V}_a); 2) the strategies of \mathcal{V}_a ; and 3) the constraints taken by the interaction relations from $\forall a_j \in \mathcal{V}_a$

to a. Now, while the extended model is used and the cyclebreaking points in G and H are fully peer to peer, a_i and $f(a_i)$ have the same three factors if G and H are isomorphic; hence, the decided strategies of a_i and $f(a_i)$ are the same.

Example 5: Now, we give an example to demonstrate Algorithm 3. Fig. 6(a) is an interaction structure with a cycle. The initial set of strategies for each agent is $\{1, ..., 100\}$. Now, a_1, a_2 , and a_3 are attributed to Group 1; thus, they can be decided by Algorithm 1. Agents a_4, a_5 , and a_6 form a cycle. We can use the following three scenarios.

- 1) If we select a_4 to break the cycle, the dependence relation c_{64} is ignored, the decided strategies are as shown in Fig. 6(c), and the usefulness degree of such decision is $U_{\rm SL} = 0.53\alpha + 0.9\beta$.
- 2) If we select a_5 to break the cycle, the dependence relation c_{45} is ignored, the decided strategies are as shown in Fig. 6(d), and the usefulness degree of such decision is $U_{\rm SL} = 0.67\alpha + 0.9\beta$.
- 3) If we select a_6 to break the cycle, the dependence relation c_{56} is ignored, the decided strategies are as shown in Fig. 6(e), now, only five dependence relations $(c_{12}, c_{13}, c_{67}, c_{78}, c_{68})$ can be satisfied, and the usefulness degree of such decision is $U_{\rm SL} = 0.62\alpha + 0.5\beta$.

From the results of the three scenarios, the decision in 2) has the maximum usefulness degree. Therefore, we should adopt such decision.

IV. CASE STUDY WITH A MULTIAGENT CITATION NETWORK

A. Introduction to Multiagent Citation Networks

In multiagent systems, the operation of some agents may be implemented by calling the operation results of other agents, which can be denoted as citation networks where the link between two nodes denotes that the agent associated with the first node directly cites the operation result of the one associated with the second node. A citation network is a typical interaction structure that is a DAG.

Now, we propose that the agents that have no direct citation links may also have indirect citation relations. Then, in a citation structure, from which does an agent cite results? Thus, we need to endow some citation rules among the agents in citation networks, which can design a strategy profile for each agent from which it can cite results.

Multiagent Citation Rules: By introducing the citation concepts in [21], we design the citation rules of agents according to the property of citation structure, shown as follows.

- 1) If there is a citation link from agent *a* to agent *b*, then *a* can cite the operation result of *b* (*Citation Rule 1*).
- 2) Agent *a* can cite the operation results of its all-orders domination agents (*Citation Rule 2*).
- 3) A direct citation link from one agent to another actually rules out a citation in the other direction. Therefore, if there is a citation link from agent *a* to agent *b*, then *b* cannot cite the result of *a* (*Citation Rule 3*).



Fig. 6. Case demonstration for Algorithm 3. (a) Interaction structure and initial strategies. (b) Social constraints of dependence relations. (c) Strategy decision result by selecting a_4 to break the cycle. (d) Strategy decision result by selecting a_5 to break the cycle. (e) Strategy decision result by selecting a_6 to break the cycle.



Fig. 7. Example for the citation network and operation citation relations.

- 4) An agent cannot cite the operation result of itself (Citation Rule 4).
- 5) Agent a cannot cite the results of agents in the all-orders dependence structures of a (Citation Rule 5).
- 6) If two agents are independent of each other in the interaction structures, then they can cite each other (Citation Rule 6).

Example 6: Fig. 7 shows an agent citation network; therefore, now, we can design some operation citation relations.

B. Strategies and Decision Making in Multiagent Citation Networks

In citation networks, the strategies of a are the set of agents from which a can cite operation results. For example, if $S_a =$ $\{a_1, a_2, a_3\}$, then agent a can cite the operation results from agents a_1, a_2 , and a_3 .

Definition 15: In the environment of citation network $\langle A, R \rangle$, where A denotes the agents and R denotes the citation links among agents, a useful decision is the one that restricts the citation relations among agents to satisfy the requirements of multiagent citation rules.

Obviously, to satisfy the requirements of citation rules $\forall a \in$ A, the set of strategies of agent a should have the following properties (decision laws).

- Law 1: ∀a ∈ A, ∑Ω_a ⊆ S_a (Citation Rule 1 and 2).
 Law 2: ∀a, b ∈ A, b ∈ ∑U_a ⇒ b ∉ S_a (Citation Rules 3 and 5).
- 3) Law 3: $\forall a \in A, a \notin S_a$ (Citation Rule 4).
- 4) Law 4: $\forall a, b \in A, a \not > b \Rightarrow (a \in S_b \land b \in S_a)$ (Citation Rule 6).

Now, according to the decision laws, we can design the conditional strategy set of agent $a \in A$ as

$$S_{a|\mathfrak{V}_a} = A - \{a\} - \sum \mathfrak{V}_a. \tag{14}$$

Therefore, the decision of the whole system can be the joint distribution of the conditional strategies of all agents which can satisfy the requirements of citation rules, i.e., we have

$$SL = S(a_1, a_2, \cdots a_n) = \bigwedge_i^n S_{a_i \mid \mho_{a_i}} = \bigwedge_i^n \left(A - \{a_i\} - \sum \mho_{a_i} \right).$$
(15)

Theorem 5: The decision making of multiagent strategies implemented by (14) and (15) is useful, and the final set of strategies available to all agents can satisfy the citation rules (i.e., can satisfy the decision laws).

Proof: Now, we prove that the decided strategies satisfy the four decision laws.

1) $\forall b \in \sum \Omega_a \Rightarrow b \notin \sum \mho_a$, we have $b \in (S_a|_{\mho_a} = A - \{a\} - \sum \mho_a)$ according to (14). Therefore, in the



Fig. 8. Example to demonstrate the conditional strategy and decision making in citation networks.

decision results determined by (15), a can cite operation results from agent $b\forall b \in \sum \Omega_a$, which then satisfies Law 1.

- 2) For agent a ∈ A, if b is the all-orders dependence agents of a, i.e., b ∈ ∑ 𝔅_a, then we have b ∉ (S_{a|𝔅_a} = A {a} ∑ 𝔅_a) according to (14). Thus, in the decision results determined by (15), a cannot cite operation results from b∀b ∈ ∑ 𝔅_a, which satisfies Law 2.
- For agent a ∈ A, we have a ∉ (S_{a|U_a} = A {a} ∑U_a) according to (14). Therefore, in the decision results determined by (15), a cannot cite operation results from itself, which satisfies Law 3.
- 4) ∀a, b ∈ A, a ≠ b ⇒ a ∉ ∑ 𝔅_b ∧ b ∉ ∑ 𝔅_a; therefore, b ∈ (S_{a|𝔅a} = A {a} ∑ 𝔅_a) ∧ a ∈ (S_{b|𝔅b}=A {b} ∑ 𝔅_b) is true. Thus, in the results decided by (15), a can cite operation results from b and vice versa, which satisfies Law 4.

Proposition 1: In the decision results determined by (14) and (15), agents having structurally equivalent interactions have the same set of strategies except for the citations between the same set of strategies except for the citations between

 $\begin{array}{ll} \text{themselves, i.e., } a \equiv b \Rightarrow ((S_{a|\mho_a} - \{b\}) = (S_{b|\mho_b} - \{a\})).\\ Proof: \ a \equiv b \Rightarrow (\sum \mho_a = \sum \mho_b). \ \text{According to (14),}\\ S_{a|\mho_a} - \{b\} = A - \{a\} - \sum \mho_a - \{b\} \ \text{and} \ S_{b|\mho_b} - \{a\} = A - \{b\} - \sum \mho_b - \{a\}. \ \text{Therefore, we have} \ S_{a|\mho_a} - \{b\} = S_{b|\mho_b} - \{a\}. \end{array}$

Example 7: We take the citation network in Fig. 8 as an example. The decision-making result is

$$SL = S(a_1, a_2, a_3, a_4, a_5)$$

= $S_{a_1} \wedge S_{a_2|a_1} \wedge S_{a_3|a_1} \wedge S_{a_4|a_2, a_3} \wedge S_{a_5|a_4}$
= $\underbrace{\{a_2, a_3, a_4, a_5\}}_{S_{a_1}} \wedge \underbrace{\{a_3, a_4, a_5\}}_{S_{a_2}} \wedge \underbrace{\{a_2, a_4, a_5\}}_{S_{a_3}}$
 $\wedge \underbrace{\{a_5\}}_{S_{a_4}} \wedge \underbrace{\{\}}_{S_{a_5}}$.

Obviously, the aforementioned SL for the citation network in Fig. 8 is useful. Moreover, $a_2 \equiv a_3$; hence, we have $S_{a2} - \{a_3\} = S_{a3} - \{a_2\} = \{a_4, a_5\}$.

C. Decision-Making Algorithm in Citation Networks

We use Algorithm 1 and (15) to make the strategy decision in citation networks, shown as Algorithm 4.

Algorithm 4. Decision making of multiagent strategies in citation networks

- Input: $A = \{a_1, a_2, \dots, a_n\}$ and interaction relations R.
- $\forall a_i \in A: S_{ai} = A;$
- *Creatstack* (stack);
- $\forall a_i \in A$: *if* $Dep_{ai} = \{ \}$, push (a_i, stack) ;
- $Int \ count = 0;$
- While [!empty(stack)] do:

 a_u = pop(stack); S_{au} = S_{au} {a_u};
 count++;
 ∀a_i ∈ Ωa_u:

 S_{temp} = A S_{au};
 S_{aj} = S_{aj} S_{temp};
 Dep_{aj} = Dep_{aj} {⟨a_u, a_j⟩};
 If Dep_{aj} = { }, then push(a_j, stack);

 If count ≠ n, then report error

 else output S_{ai} ∀a_i ∈ A.

Example 8: Three-dimensional citation cube. We use the citation cube in Fig. 9 to demonstrate Algorithm 4. At first, we decide the strategies of agents d and f that have no dependence substructures. The progress of decision making is shown by (ii)–(ix) in Fig. 9.

D. On the Adjustment and Scalability in Citation Networks

1) Adjustment for the Oscillation of Citation Links: Decision making should be adjusted as a choice for the interaction oscillation regarding the history. In the operations of agent systems, some new citation links may be added to the structure and some old citation links may be deleted from the existing structure. We do not need to decide the entire strategies from the beginning, as that would be costly. Instead, we should adjust the existing decided strategies locally.

Given that a citation network $N = \langle A, R \rangle$ and N is a DAG, if an existing relation $\langle a_u, a_v \rangle$ is deleted from N, then the new structure $N' = \langle A, R' \rangle$, $R' = R - \{\langle a_u, a_v \rangle\}$, is still a DAG. However, if a new citation link is added to the citation network, we need to justify whether there are any cycles in the structure.

While some citation links oscillate in the citation network, we can adjust the decided strategies according to the following adjustment law.

Adjustment Law:

Given an environment ⟨A, R⟩, a_u, a_v ∈ A, if a new citation link ⟨a_u, a_v⟩ is added to the citation network and does not produce any cycles, then the set of strategies ∀a_i ∈ ({a_v} ∪ ∑Ω'_{a_v}) can be changed as follows:

$$\forall a_i \in \left(\{a_v\} \cup \sum \Omega'_{a_v}\right), \quad S'_{a_i} = S_{a_i} - \left(A - S_{a_u}\right) \quad (16)$$

where $\sum \Omega'_{a_v}$ are the all-orders domination agents of a_v in the new structure, S_{ai} is the set of strategies of agent a_i in the old structure, and S'_{a_i} is the set of strategies of agent a_i in the new structure.



Fig. 9. Case demonstration for the decision making in citation networks.

Moreover, the strategies of $\{a_i | \forall a_i \in (A - (\{a_v\} \cup \sum \Omega'_{a_v}))\}$ do not need to be changed.

Given an environment ⟨A, R⟩, a_u, a_v ∈ A, if an existing citation link ⟨a_u, a_v⟩ is deleted, then the set of strategies ∀a_i ∈ ({a_v} ∪ ∑Ω'_{a_v}) can be changed as follows:

$$\forall a_i \in \left(\{a_v\} \cup \sum \Omega'_{a_v}\right), \\ S'_{a_i} = S_{a_i} \cup \left(\{a_u\} \cup \sum \mathfrak{I}'_{a_u} - \sum \mathfrak{I}'_{a_i}\right) \quad (17)$$

where $\sum \Omega'_{a_v}$, $\sum \mathcal{U}'_{a_u}$, $\sum \mathcal{U}'_{a_i}$ are the ones in the new structure, S_{ai} is the set of strategies of a_i in the old structure, and S'_{a_i} is the set of strategies of agent a_i in the new structure.

Moreover, the strategies of $\{a_i | \forall a_i \in (A - (\{a_v\} \cup \sum \Omega'_{a_v}))\}$ need not be changed.

Theorem 6: Given an environment $\langle A, R \rangle$ and a decision SL that is useful for the existing citation structure, the adjustment law can obtain useful decision for the new citation structure.

Proof: The proof can be seen in the Appendix. \Box *Example 9:* We can take the citation network and strategies in Fig. 9 (ix) as an example to demonstrate our adjustment law. Fig. 10 shows the adjustment for interaction relation oscillation.

2) Scalability for the Growth of Citation Structures: The growth of citation networks can be based on the dynamics of interacting links that is motivated by the joining agents to



Fig. 10. Case demonstration for the adjustment law: (a) $\langle f, d \rangle$ is added to the citation structure; now, the strategies of agent d, b, c, h, g are adjusted. (b) $\langle d, h \rangle$ is deleted from the citation structure; now, the strategies of agent h and g are adjusted.

construct links (re)directing them toward selected existing agents [21]. If the citation structure at time t is $\langle A_t, R_t \rangle$, where A_t denotes the set of agents and R_t denotes the set of agent citation links, and if the citation structure at time t + iis $\langle A_{t+i}, R_{t+i} \rangle$, where A_{t+i} denotes the set of agents and R_{t+i} denotes the set of agent citation links, then the growth of agent citation structure satisfies the following: $A_t \subseteq A_{t+i}$ and $R_t \subseteq R_{t+i}$. When the citation structure grows, we do not need to make decisions by starting from scratch, which is costly. Thus, we should expand the existing decided strategies locally.

Growth Law: Given an environment $\langle A, R \rangle$ and the existing decision SL, we let an agent a and some citation links associated with it be added to the existing structure, the new set of agents be A', and the new citation structure be R'. The growth of citation links should not produce any cycles in the new citation network; now, we change the strategies of agents according to the following laws.

- 1) $\forall a_i \in \sum \mho_a$, their strategies are changed as $S'_{a_i} = S_{a_i} \cup$
- 2) $\forall a_i \in \sum_{\alpha} \Omega_a$, their strategies are changed as $S'_{a_i} = S_{a_i} S_{a_i}$
- {a} = S_{a_i}.
 3) ∀a_i ∈ (A' ∑Ω_a ∑U_a {a}) ⇒ a_i ≠ a, their strategies are changed as S'_{a_i} = S_{a_i} ∪ {a}.
 4) For agent a, S'_a = A' {a} ∑U_a.

Theorem 7: Obviously, the four parts of the growing law are all determined according to (14) and (15); thus, the growth law can obtain useful decision results.

Example 10: We now take Fig. 9 (ix) as an example; let a new agent z and two new citation links $\langle z,g\rangle$ and $\langle h,z\rangle$ be added to the structure. Now, we can change the decided strategies of the system according to our growth law; the result



Fig. 11. Case demonstration for the growth law.

is shown in Fig. 11. Obviously, the final decided strategies can satisfy the citation rules for the new citation structure; hence, the decision is useful.

V. RELATED WORK

Our research is related to the decision making of multiagents, where each agent should make decisions about which action to perform to ensure a good joint action for the whole multiagent group. Generally, related work can be categorized as follows.

1) Decision Making of Multiagents Based on Game Theory and Economics [22]: While agents inhabit a shared environment, they negotiate with each other to decide their actions [30]-[34]. To conduct negotiations, they always adopt game theory or other economics techniques, such as bargaining, auction, contracting, etc. The negotiation protocols and decisionmaking procedures are always focused. The related works include two aspects: cooperative agents and self-interested agents.

In the decision making of cooperative agents, the agents need to cooperate with each other to solve a problem or to reach a common goal. For example, Moehlman et al. [23] use decentralized negotiation to solve the distributed planning problem; Lander and Lesser [24] employ multistage negotiation as a means to conduct distributed searches among agents; Pelta and Yager [25] consider a problem of mediated group decision making where a number of agents provide a preference function over a set of alternatives and present an optimization approach for the decision strategies in mediated multiagent negotiations. Another typical example for the decision making of cooperative multiagents is the one in robot soccer, where the agents share a common decision-making criterion and take into account what their partners are able to do [26]. Therefore, in cooperative agents, they always negotiate to reach an agreement, and the decision is made according to the maximum utility of the system.

In the decision making of self-interested agents, the agents try to maximize payoff without concern of the global good; thus, such a self-interested agent will choose the best negotiation strategy for itself [27]. Game theory is a branch of economics that is always used to study interactions between self-interested agents [3]. Game theory may be used to analyze the problems of how interaction strategies can be designed to maximize the welfare of an agent in a multiagent encounter and how protocols or mechanisms can be designed that have certain desirable properties [2]. An agent's equilibrium strategy depends on the information that it has about the preferences and behaviors of other agents. The decision making of self-interested agents is typically seen in the market or electronic commerce [28]. For example, Lomuscio et al. [29] present a classification scheme for the negotiation of self-interested agents in electronic commerce.

2) Decision Making of Networked Agents: With large-scale and networked application environments, distributed decision making for the coordination of networked agents has received much attention in recent years. In the related works on decision making of networked agents, a network of agents with initially different opinions can reach a collective decision and hence take action in a distributed manner [11]. Saber and Murray provide convergence, performance, and robustness analyses of an agreement protocol for a network of integrator agents with directed information flow and (perhaps) switching topology, which mainly relies on the tools of algebraic graph theory and matrix theory [10]. Roy et al. [11] introduce a quasi-linear stochastic distributed protocol that can be used by a network of sensing agents to reach a collective action agreement; moreover, they put forth the viewpoint that it is useful to consider the information-fusion and decision-making tasks of networks with sensing agents jointly, as a decentralized stabilization or agreement problem. Gal et al. [34] describe several new decision-making models that represent, learn, and adapt to various social attributes that influence people's decision making in open mixed networks including agents and people.

3) Modeling the Interdependence Among Multiagents: The dependence among multiagents can be modeled by dependence networks. The dependence network can be used for the study of emerging social structures, such as groups and collectives, and may form a knowledge base for managing complexity in both competitive and organizational or other cooperative contexts [12]. Sichman and Conte [12] model multiagent interdependences among different agents' goals and actions and construct a tool for predicting and simulating their emergence. Wong and Butz [13] propose an automated process for constructing the combined dependence structure of a multiagent probabilistic network, where the dependence structure is a graphical representation of the conditional independencies that are known to hold in the problem domain. Generally, the related works on the interdependence among multiagents mainly focus on the knowledge representation and reasoning dependence among multiagents.

Summarization: The main concerns of related works can be summarized as follows: 1) In the previous decision-making works of multiagents based on game theory and economics, they mainly focus on negotiation protocols and decisionmaking procedures; 2) in the previous works on the decision making of networked agents, they mainly concern the agreement problems in which all agents in the network must achieve the same opinion and on the connection between the network topology and the performance of the agreement protocol; and 3) in the previous works on the interdependence among multiagents, they mainly focus on the knowledge representation and reasoning dependence among multiagents.

Therefore, previous works seldom take into account the interaction structures of agents. Aiming to solve the structured interaction collision problem of networked multiagents, this paper investigates the interaction structure-oriented decision making.

VI. CONCLUSION

Networked structures are very popular in the large-scale multiagent systems. We have presented a novel interactionstructure-oriented decision model of networked multiagent strategies, which can satisfy the requirement of interaction structure among agents. The presented model can restrict the strategies of all agents to match their interaction positions. The presented decision-making model contains two parts: One is the basic model for the directed acyclic interaction structure and the other is the extended model for the directed interaction structure with cycles. We theoretically prove that the former can produce the unique outcome, which is to minimize the conflicts among agents, and that the latter can produce the maximum utility. Moreover, the model can produce the same resulted strategies as for isomorphic structures.

Finally, we adopted a multiagent citation network to make a case study. Through the case study, we can then see that our model can minimize collisions for citation relations. In our case study, citation networks are considered DAGs. However, there are also some other special cases of citation structures that are not DAGs, such as mutually citation companion agents and cyclic citation structures occurred in some agents; therefore, we will solve the strategy decision in cyclic citation structures by using our extended model. Moreover, in the future, we will focus on the application of our decision-making model in more complex interaction structures, such as hypergraph, complex social networks, etc.

APPENDIX **PROOF OF THEOREM 6**

Obviously, the adjustment law can result in a useful decision only if we can prove that the adjustment law satisfies the requirements of the decision laws in Section IV-B.

- 1) Law 1: $\forall a \in A, \sum \Omega_a \subseteq S_a$.

 - a) $\forall a_i \in (A \{a_u, a_v\} \sum \Omega'_{a_v})$ i) $\forall a_j \in (A \{a_u, a_v\} \sum \Omega'_{a_v})$, if $a_j \in \sum \Omega_{ai}$, then $a_j \in \sum \Omega'_{a_i}$ and $a_j \in S_{a_i}$. Now, $S'_{a_i} = S_{a_i}$; hence, $a_j \in S'_{a_i}$.
 - ii) For a_u , if $a_u \in \sum \Omega_{a_i}$, then $a_u \in \sum \Omega'_{a_i}$ and $a_u \in S_{a_i}$. Now, $S'_{a_i} = S_{a_i}$; thus, $a_u \in S'_{a_i}$. iii) For $\forall a_j \in (\{a_v\} \cup \sum \Omega'_{a_v})$, if $a_j \in \sum \Omega_{a_i}$, then
 - $a_j \in S_{a_i}$.
 - When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following occurs.

 $a_j \in \Omega'_{a_i}$ and $S'_{a_i} = S_{a_i}$; thus, we have $a_j \in$ S'_{a_i} .

• When an old citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following occurs.

If $a_j \in \sum \Omega'_{a_i}$, $S'_{a_i} = S_{a_i}$; therefore, we have $a_j \in S'_{a_i}$. If $a_j \notin \sum \Omega'_{a_i}$, then $a_j \notin a_i$; therefore, we also have $a_i \in S'_{a_i}$.

- b) For agent a_u
 - i) $\forall a_j \in (A \{a_u, a_v\} \sum \Omega'_{a_v}), \text{ if } a_j \in \sum \Omega_{a_u},$ then $a_j \in \sum \Omega'_{a_u}$ and $a_j \in S_{a_u}$. Now, $S'_{a_u} = S_{a_u}$; thus, $a_j \in \overline{S'_{a_u}}$. ii) $\forall a_j \in (\{a_v\} \cup \sum \Omega'_{a_v})$, if $a_j \in \sum \Omega_{a_u}$, then $a_j \in \sum \Omega_{a_v}$.
 - S_{a_u} .
 - When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following occurs.

Now that $a_j \in \sum \Omega'_{a_u}$ and $S'_{a_u} = S_{a_u}$ is achieved, we have $a_j \in S'_{a_u}$. • When an old citation link $\langle a_u, a_v \rangle$ is deleted

from the citation structure, the following occurs.

If $a_j \in \sum \Omega'_{a_u}$, then $S'_{a_u} = S_{a_u}$; hence, we have $a_j \in S'_{a_u}$. If $a_j \notin \sum \Omega'_{a_u}$, then $a_j \not \approx a_u$; therefore, we have $a_j \in S'_{a_u}$.

- c) $\forall a_i \in (\{a_v\} \cup \sum \Omega'_{a_v})$ i) $\forall a_j \in (A \sum \Omega'_{a_v}), a_j \notin \sum \Omega'_{a_i}$; hence, we do not need to address them.
 - ii) $\forall a_j \in \sum \Omega'_{a_v}$, if $\forall a_j \in \sum \Omega_{a_i}$, then $\forall a_j \in \sum \Omega'_{a_i}$, $a_j \in S_{a_i}$.
 - When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following occurs. Now, $S'_{a_i} = S_{a_i} - (A - S_{a_u})$ and $a_j \in$ S_{a_u} ; hence, $a_j \in S'_{a_i}$.
 - When an old citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following is possible: $S'_{a_i} \supseteq S_{a_i}$; thus, we have $a_j \in S'_{a_i}$.

2) Law 2: $\forall a, b \in A, b \in \sum \mathfrak{V}_a \Rightarrow b \notin S_a$.

- a) $\forall a_i \in (A \{a_v\} \sum \Omega'_{a_v})$ i) $\forall a_j \in (A - \{a_v\} - \sum_{i=1}^{u_v} \Omega'_{a_v}), \text{ if } a_j \in \sum_{i=1}^{u_v} \Im_{a_i}, \text{ then }$ $a_j \notin S_{a_i}$. Now, $a_j \in \sum O'_{a_i}$ and $S'_{a_i} = S_{a_i}$; therefore, we have $a_j \notin S'_{a_i}$.
 - ii) $\forall a_j \in (\{a_v\} \cup \sum \Omega'_{a_v}), a_j \notin \sum \mho_{a_i};$ hence, we need not address them.
- $\begin{array}{l} \mathsf{b}) \ \forall a_i \in (\{a_v\} \cup \sum \Omega'_{a_v}) \\ \mathsf{i}) \ \forall a_j \in A, \text{if } a_j \in \sum \mho_{a_i}, \text{then } a_j \not\in S_{a_i}. \end{array}$
 - When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following happens.

Now, $a_j \in \sum U'_{a_i}, S'_{a_i} = S_{a_i} - (A - S_{a_u}),$ and $a_j \notin S_{a_i}$; hence, we have $a_j \notin S'_{a_i}$.

• When an existing citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following occurs.

If $a_j \notin \sum \mho'_{a_i}$, we need not address them. If $a_j \in \overline{\sum} \mho'_{a_i}$, now, $S'_{a_i} = S_{a_i} \cup (\{a_u\} \cup \sum \mho'_{a_u} - \sum \mho'_{a_i})$ and $a_j \notin S_{a_i}$; thus, we have $a_j \notin S'_{a_i}$.

3) Law 3: $\forall a \in A, a \notin S_a$.

- a) $\forall a_i \in (A \{a_v\} \sum \Omega'_{a_v}), a_i \notin S_{a_i}$. Now, $S'_{a_i} = S_{a_i}$; hence, $a_i \notin S'_{a_i}$.
- b) $\forall a_i \in (\{a_v\} \cup \sum \tilde{\Omega}'_{a_v}), a_i \notin S_{a_i}.$
 - i) When a new citation link $\langle a_u, a_v \rangle$ is added to the interaction structure, the following occurs.

Now,
$$S'_{a_i} = S_{a_i} - (A - S_{a_u}), a_i \notin S_{a_i}$$
; hence,
 $a_i \notin S'_{a_i}$.

ii) When an old citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following happens.

Now, $S'_{a_i} = S_{a_i} \cup (\{a_u\} \cup \sum U'_{a_u} - \sum U'_{a_i}), a_i \notin S_{a_i}, a_i \notin \sum U'_{a_u}$; therefore, $a_i \notin S'_{a_i}$.

4) Law 4: $\forall a, b \in A, a \not > b \Rightarrow (a \in S_b \land b \in S_a).$

- 1) $\forall a_i \in (A \{a_v\} \sum \Omega'_{a_v})$ i) $\forall a_j \in (A - \{a_v\} - \sum \Omega'_{a_v})$, if $a_i \not > a_j$ is in the old citation structure, then $a_i \in S_{a_j} \land a_j \in S_{a_i}$ is true. Now, it is true that $a_i \not > a_j$ is in the new citation structure and $S'_{a_j} = S_{a_j} \wedge S'_{a_i} = S_{a_i}$; therefore, we have $a_i \in S'_{a_i} \land a_j \in S'_{a_i}$.
 - ii) $\forall a_j \in (\{a_v\} \cup \sum \Omega'_{a_v})$, if $a_i \not > a_j$ is in the old citation structure, then $a_i \in S_{a_i} \land a_j \in S_{a_i}$ is true.
 - When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following happens.

Now, $S'_{a_j} = S_{a_i} \Rightarrow a_j \in S'_{a_i}$ and $S'_{a_j} = S_{a_j} - (A - S_{a_u})$. If $a_i \not\models a_j$ is in the new citation structure, then $a_i \not > a_u$, so $a_i \in S_{a_u}$. Thus, we have $a_i \in S'_{a_i}$.

• When an existing citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following takes place.

If $a_i \not > a_j$ is in the old citation structure, then $a_i \not > a_j$ is also true in the new citation structure. Now, $S'_{a_i} = S_{a_i} \Rightarrow a_j \in$ $\begin{array}{l} S'_{a_i}. \ S'_{a_j} = S_{a_j} \cup (\{a_u\} \cup \sum \mho'_{a_u} - \sum \mho'_{a_j}),\\ a_i \not \models a_j \Rightarrow a_i \not \in \sum \mho'_{a_j}, \text{ and } a_i \in S_{a_j}; \text{ thus, we} \end{array}$ have $a_i \in S'_{a_i}$.

- 2) $\forall a_i \in (\{a_v\} \cup \sum \Omega'_{a_v})$ i) $\forall a_j \in (A - \{a_u, a_v\} - \sum \Omega'_{a_v})$
 - When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following process is made possible.

If $a_i \not > a_j$ is in the old citation structure, then we have $a_i \in S_{a_j} \land a_j \in S_{a_i}$. We have $S'_{a_j} = S_{a_j} \Rightarrow a_i \in S'_{a_j}$. Now, if $a_i \not > a_j$ is in the new citation structure, then $a_i \not\models$ $a_j \Rightarrow a_j \not\in \sum \mho'_{a_i}$ and $S'_{a_i} = S_{a_i} \cup (\{a_u\} \cup$ $\sum_{i=1}^{n} \mathcal{U}'_{a_{u}} - \sum_{i=1}^{n} \mathcal{U}'_{a_{i}}$. Consequently, we have $a_{j} \in S'_{a_{i}}$.

If $a_i \in \sum \Omega_{a_i}$, then $a_i \not > a_j$ is not true in the new citation structure; therefore, there is no need for us to address them.

• When an existing citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following occurs.

If $a_i \not > a_j$ or $a_i \in \sum \Omega_{a_j}$ is in the old citation structure, then we have $a_i \in S_{a_i}$. We have $S'_{a_i} = S_{a_j} \Rightarrow a_i \in S'_{a_j}$. Now, if $a_i \not > a_j$ is in the new citation structure, then $a_i \not > a_j \Rightarrow$ $\begin{array}{l} a_{j} \not\in \sum \mho'_{a_{i}} \land a_{j} \in \sum \mho'_{a_{u}} \ \, \text{and} \ \, S'_{a_{i}} = S_{a_{i}} \cup \\ (\{a_{u}\} \cup \sum \mho'_{a_{u}} - \sum \mho'_{a_{i}}). \ \, \text{Thus, we have} \end{array}$ $a_j \in S'_{a_i}$.

- ii) For agent a_u
 - When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following happens.
 - Obviously, $a_i \not \models a_j$ is not true in the new structures; hence, we need not address them.
 - When an existing citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following occurs.

If $a_i \not\models a_u$ or $a_i \in \sum \Omega_{a_u}$ is in the old citation structure, then we have $a_i \in S_{a_u}$. We have $S'_{a_u} = S_{a_u} \Rightarrow a_i \in S'_{a_u}$. Now, if $a_i \not\models a_u$ is in the new citation structure, then $a_i \not\models a_u \Rightarrow a_u \notin \sum \mathcal{U}'_{a_i}$ and $S'_{a_i} = S_{a_i} \cup (\{a_u\} \cup \sum \mathcal{U}'_{a_u} - \sum \mathcal{U}'_{a_i})$. Hence, we have $a_u \in S'_{a_i}$. iii) $\forall a_j \in (\{a_v\} \cup \sum \Omega'_{a_v})$

• When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following takes place.

As the citation structure between a_i and a_j is not changed, if $a_i \not > a_u$ is in the new citation structure, then $a_i \not > a_u$ is also true in the old citation structure and $a_i \in S_{a_j} \land a_j \in S_{a_i}$. Now, $S'_{a_i} = S_{a_i} - (A - S_{a_u})S'_{a_j} = S_{a_j} - (A - S_{a_u})$, $a_i \notin (A - S_{a_u})$, and $a_j \notin (A - S_{a_u})$; thus, we have $a_i \in S'_{a_j} \land a_j \in S'_{a_i}$.

 When an existing citation link (a_u, a_v) is deleted from the citation structure, the following scenario occurs.

Since the citation structure between a_i and a_j is not changed and if $a_i \not\models a_u$ is in the new citation structure, then $a_i \not\models a_u$ is also true in the old citation structure and $a_i \in S_{a_j} \land a_j \in S_{a_i}$. Now, $S'_{a_i} \supseteq S_{a_i}$ and $S'_{a_j} \supseteq S_{a_j}$; then, we have $a_i \in S'_{a_j} \land a_j \in S'_{a_i}$.

REFERENCES

- B. Edmonds, "Towards a descriptive model of agent strategy search," *Comput. Econ.*, vol. 18, no. 1, pp. 113–135, 2001.
- [2] S. Parsons and M. Wooldridge, "Game theory and decision theory in multi-agent systems," *Auton. Agents Multi-Agent Syst.*, vol. 5, no. 3, pp. 243–254, Sep. 2002.
- [3] N. R. Jennings, P. Faratin, A. R. Lomuscio, S. Parsons, M. J. Wooldridge, and C. Sierra, "Automated negotiation: Prospects, methods and challenges," *Group Decision Negotiation*, vol. 10, no. 2, pp. 199–215, Mar. 2001.
- [4] K. Sarit, Strategic Negotiation in Multiagent Environments. Cambridge, MA: MIT Press, 2001.
- [5] S. Abdallah and V. Lesser, "Multiagent reinforcement learning and selforganization in a network of agents," in *Proc. 6th Int. Joint Conf. AAMAS*, Honolulu, Hawaii, May 2007, pp. 172–179.
- [6] Y. Jiang and J. Jiang, "Contextual resource negotiation-based task allocation and load balancing in complex software systems," *IEEE Trans. Parallel Distrib. Syst.*, vol. 20, no. 5, pp. 641–653, May 2009.
- [7] L. Cao, C. Zhang, and M. Zhou, "Engineering open complex agent systems: A case study," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 38, no. 4, pp. 483–496, Jul. 2008.
- [8] S. Wasserman and K. Faust, Social Network Analysis: Methods and Applications. Cambridge, U.K.: Cambridge Univ. Press, 1997.

- [9] O. Pacheco and J. Carmo, "A role based model for the normative specification of organized collective agency and agents interaction," *Auton. Agents Multi-Agent Syst.*, vol. 6, no. 2, pp. 145–184, Mar. 2003.
- [10] R. O. Saber and R. M. Murray, "Agreement problems in networks with directed graphs and switching topology," in *Proc. 42nd IEEE Conf. Deci*sion Control, Honolulu, HI, Dec. 9–12, 2003, pp. 4126–4132.
- [11] S. Roy, K. Herlugson, and A. Saberi, "A control-theoretic approach to distributed discrete-valued decision-making in networks of sensing agents," *IEEE Trans. Mobile Comput.*, vol. 5, no. 8, pp. 945–957, Aug. 2006.
- [12] R. Conte and J. S. Sichman, "Dependence graphs: Dependence within and between groups," *Comput. Math. Org. Theory*, vol. 8, no. 2, pp. 87–112, Jul. 2002.
- [13] S. K. M. Wong and C. J. Butz, "Constructing the dependency structure of a multiagent probabilistic network," *IEEE Trans. Knowl. Data Eng.*, vol. 13, no. 3, pp. 395–415, May 2001.
- [14] Y. Shoham and M. Tennenholtz, "On the emergence of social conventions: Modeling, analysis and simulations," *Artif. Intell.*, vol. 94, no. 1/2, pp. 139–166, Jul. 1997.
- [15] G. Boella and L. van der Torre, "The evolution of artificial social systems," in *Proc. 19th IJCAI*, Edinburgh, U.K., Aug. 5, 2005, pp. 1655–1656.
- [16] J. A. Bondy and U. S. R. Murty, *Graph Theory and Applications*. London, U.K.: Macmillan, 1976.
- [17] B. R. Preiss, Data Structures and Algorithms with Object-Oriented Design Patterns in C++. New York: Wiley, 1999.
- [18] G. Chartrand, "Isomorphic graphs," in *Introductory Graph Theory*. New York: Dover, 1985, pp. 32–40, §2.2.
- [19] S. Skiena, "Graph isomorphism," in *Implementing Discrete Mathematics: Combinatorics and Graph Theory with Mathematica*. Reading, MA: Addison-Wesley, 1990, pp. 181–187, §5.2.
- [20] R. Diestel, Graph Theory, 3rd ed. Heidelberg, Germany: Springer-Verlag, 2005.
- [21] S. Lehmann, B. Lautrup, and A. D. Jackson, "Citation networks in high energy physics," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 68, no. 2, p. 026 113(1–8), Aug. 2003.
- [22] S. Kraus, "Automated negotiation and decision making in multiagent environments," *Lecture Notes in Artificial Intelligence*, vol. 2086, pp. 150– 172, 2001.
- [23] T. Moehlman, V. Lesser, and B. Buteau, "Decentralized negotiation: An approach to the distributed planning problem," *Group Decision Negotiation*, vol. 1, no. 2, pp. 161–191, Aug. 1992.
- [24] S. E. Lander and V. R. Lesser, "Customizing distributed search among agents with heterogeneous knowledge," in *Proc. 1st Int. Conf. Inf. Knowl. Manage.*, Baltimore, MD, 1992, pp. 335–344.
- [25] D. A. Pelta and R. R. Yager, "Decision strategies in mediated multiagent negotiations: An optimization approach," *IEEE Trans. Syst.*, *Man, Cybern. A, Syst., Humans*, vol. 40, no. 3, pp. 635–640, May 2010.
- [26] P. Guerrero, J. Ruiz del Solar, M. Romero, and L. Herrera, "An integrated multi-agent decision making framework for robot soccer," in *Proc. 6th LARS*, Valparaíso, Chile, Oct. 29–30, 2009, pp. 1–6.
- [27] T. W. Sandholm, "Negotiation among self-interested computationally limited agents," Electronic Doctoral Dissertations, Univ. Massachusetts, Amherst, MA, Jan. 1, 1996, Paper AAI9709647.
- [28] T. Sandholm, "Agents in electronic commerce: Component technologies for automated negotiation and coalition formation," *Auton. Agents Multi-Agent Syst.*, vol. 3, no. 1, pp. 73–96, Mar. 2000.
- [29] A. R. Lomuscio, M. Wooldridge, and N. R. Jennings, "A classification scheme for negotiation in electronic commerce," *Group Decision Negotiation*, vol. 12, no. 1, pp. 31–56, Jan. 2003.
- [30] Y. Jiang, "Concurrent collective strategy diffusion of multiagents: The spatial model and case study," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 39, no. 4, pp. 448–458, Jul. 2009.
- [31] Y. Jiang, "Extracting social laws from unilateral binary constraint relation topologies in multiagent systems," *Expert Syst. Appl.*, vol. 34, no. 3, pp. 2004–2012, Apr. 2008.
- [32] Y. Jiang and T. Ishida, "A model for collective strategy diffusion in agent social law evolution," in *Proc. 20th IJCAI*, Hyderabad, India, Jan. 6–12, 2007, pp. 1353–1358.
- [33] Y. Jiang, J. Jiang, and T. Ishida, "Agent coordination by trade-off between locally diffusion effects and socially structural influences," in *Proc. 6th Int. Conf. AAMAS*, Honolulu, HI, May 14–18, 2007, pp. 520–522.
- [34] Y. Gal, B. Grosz, S. Kraus, A. Pfeffer, and S. Shieber, "Agent decisionmaking in open mixed networks," *Artif. Intell.*, vol. 174, no. 18, pp. 1460– 1480, Dec. 2010.



Yichuan Jiang (M'07) received the Ph.D. degree in computer science from Fudan University, Shanghai, China, in 2005.

From July to October 2005, he was a Postdoctoral Researcher with the Department of Computer Science, Hong Kong Baptist University, Kowloon, Hong Kong. He was then a Japan Society for the Promotion of Science Research Fellow with the Department of Social Informatics, Kyoto University, Kyoto, Japan. He is currently a Professor with the Laboratory of Complex Systems and Social Computing, School of

Computer Science and Engineering, Southeast University, Nanjing, China. His current research interests include multiagent systems, complex distributed systems, social networks, and social computing. He has authored or coauthored more than 60 scientific articles published in refereed journals and conferences, such as the IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS, the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART C: APPLICATIONS AND REVIEWS, the *Journal of Parallel and Distributed Computing*, the International Joint Conference on Artificial Intelligence, and the International Conference on Autonomous Agents and Multiagent Systems. He is also a member of editorial board of *Advances in the Internet of Things* and of the *Chinese Journal of Computers*.

Dr. Jiang is a member of the IEEE Computer Society and the Association for Computing Machinery and a senior member of China Computer Federation and The Chinese Institute of Electronics. He was awarded the New Century Excellent Talents in University of State Education Ministry of China award, the Best Paper Award from PRIMA2006, the Nomination Award for the National Excellent Doctoral Dissertation of China, and the Young Distinguished Professor award from Southeast University. He is also the recipient of the HwaYing Young Scholar Award in 2008 from The HwaYing Education and Culture Foundation.



Jing Hu received the Ph.D. degree from Nanjing University, Nanjing, China, in 2005.

She is currently an Associate Professor with the College of Arts, Nanjing University of Aeronautics and Astronautics, Nanjing. Her current research interests include multiagent-based virtual drama, multiagent modeling for arts, and mind-art computation.



Donghui Lin received the Ph.D. degree in informatics from Kyoto University, Kyoto, Japan, in 2008.

Since then, he has been a Researcher with the Knowledge Creating Communication Research Center, National Institute of Information and Communications Technology, Tokyo, Japan. He has authored or coauthored more than 20 scientific articles published in refereed journals and conferences. His current research interests include multiagent systems and services computing.